Since the tangent modulus stiffness a is positive, it is evident that, as n increases, the value of $\bar{x}^{(n)}$ approaches the exact value given by Eq. (5).

Reference

[1] Marcal, P. V., "A Comparative Study of Numerical Methods of Elastic-Plastic Analysis," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 157–158.

Reply by Author to M. A. Salmon

P. V. Marcal*

Brown University, Providence, R. I.

THE lack of convergence of the constant strain approach for an elastic perfectly plastic material was argued for a two-dimensional constant stress element.[1] It is not possible to use Salmon's example of a one-dimensional constant stress element to test the above claim, since the material exhibits different behavior in the two different cases. Because of the normal flow rule of plasticity, the yielded material in two dimensions still possesses a certain amount of resistance to straining. This is reflected in the stress-strain relation $[P^-]$. This is not true in the one-dimensional case where $[P^-]$ is equal to zero.

Salmon's criticism does, in fact, raise an important point. Since most of the analysis was developed in general matrix form, it would be expected that, for an elastic perfectly plastic material, the lack of convergence should apply equally to all types of constant stress elements. However, this line of reasoning neglects the fact that $[P^-]$ does not exist for a one-dimensional truss element. Because $[P^-]$ does not exist, Eq. (13) of Ref. 1 is invalid, and the convergence study that is based on this equation can no longer be expected to hold.

Reference

[1] Marcal, P. V., "A Comparative Study of Numerical Methods of Elastic-Plastic Analysis," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 157–158.

Received February 26, 1968.

* Assistant Professor, Division of Engineering. Member AIAA.

Comment on "A Formula for Updating the Determinant of the Covariance Matrix"

James E. Potter* and Donald C. Fraser†

Massachusetts Institute of Technology, Cambridge, Mass.

A FORMULA was presented in Ref. 1 for updating the determinant of the covariance matrix of state estimation errors when measurement statistics are incorporated. This result may be obtained in a much simpler way if a more general identity is first proved. This general identity is

$$|A||D + CA^{-1}B| = |D||A + BD^{-1}C| \tag{1}$$

where the individual matrices have the following form:

$A = n \times n$ (nonsingular) $\quad C = m \times n$

$B = n \times m \quad D = m \times m$ (nonsingular)

This identity is obtained by manipulating partitioned matrices as follows:

$$\begin{bmatrix} A & -B \\ C & D \end{bmatrix} \begin{bmatrix} I & A^{-1}B \\ O & I \end{bmatrix} = \begin{bmatrix} A & O \\ C & (D + CA^{-1}B) \end{bmatrix} \tag{2}$$

Since the second matrix on the left side of Eq. (2) has unity determinant, there results

$$\begin{vmatrix} A & -B \\ C & D \end{vmatrix} = |A|\ |D + CA^{-1}B| \tag{3}$$

. Similarly,

$$\begin{bmatrix} A & -B \\ C & D \end{bmatrix} \begin{bmatrix} I & O \\ -D^{-1}C & I \end{bmatrix} = \begin{bmatrix} A + BD^{-1}C & B \\ O & D \end{bmatrix} \tag{4}$$

$$\begin{vmatrix} A & -B \\ C & D \end{vmatrix} = |D|\ |A + BD^{-1}C| \tag{5}$$

Equating Eqs. (3) and (5) gives the identity of Eq. (1). Using the following change of vocabulary in Eq. (1):

$$A = P'^{-1} \qquad B = H \qquad C = H^T \qquad D = R \tag{6}$$

together with the optimum linear filter update equation

$$P^{-1} = P'^{-1} + HR^{-1}H^T \tag{7}$$

leads directly to the main result of Ref. 2,

$$|P|/|P'| = |R|/|R + H^TP'H| \tag{8}$$

By making the substitutions

$$D = I \qquad C = DC \tag{9}$$

Eq. (1) can be extended to the case where D is singular. The result is

$$|A|\ |I + DCA^{-1}B| = |A + BDC| \tag{10}$$

Reference

[1] Potter, J. E. and Fraser, D. C., "A Formula for Updating the Determinant of the Covariance Matrix," *AIAA Journal*, Vol. 5, No. 7, July 1967, pp. 1352–1354.

Received March 28, 1968.

* Assistant Professor, Dept. of Aeronautics and Astronautics. Member AIAA.

† Staff Member, Instrumentation Laboratory.

Comment on "Feasibility of a High-Performance Aerodynamic Impulse Facility"

Clarence J. Harris*

General Electric Company, Valley Forge, Pa.

THE entropy correlation of the nonequilibrium chemical species concentrations of expanding air offered in Refs. 1 and 2 was based upon l values [i.e., $A/A^* = 1 + (x/l)^2$] of only 1 and 4.74 cm. Therefore, this correlation as reproduced in Ref. 3 is in error. The value of $l = 10$ cm shown in Ref. 3 appears to be a typographical error introduced when

Received March 7, 1968.

* Specialist, Physics, Fluid Dynamics.